Crate World Apprentice Notebook

Congratulations! You have been accepted as a Crate World Apprentice.

Our job at Crate World is to move crates. Some people don't realize the importance of moving crates. Our crates keep the world going!

Our motto is: ***Move Those Crates!***

You'll understand the Crate World Apprentice Notebook more if you try the activities, THEN look at the notes if you want more information.

But you do need pencil and paper! You will be moving numbers around to figure out routes. You can move numbers around on paper to make them easier to work with. If you only move numbers with your mind, you are likely to make more mistakes. Your boss does not want lost crates (and neither should you.)

It's much easier to be a Crate World Apprentice (and to learn math) if you're trying to work with numbers in different ways. If something works, you know you're on the right track. If something doesn't work, that's good information too.

Doing things with numbers is a puzzle. Everything in math was new to those who discovered it at one time. They had to figure it out. You're figuring it out now.

You also can work with a friend or ask a boss-type person like a parent or teacher. You could even look in a good math textbook or at one of the math websites that explain things well.

And remember --- **it's easier with math!**

Apprentice 1 Notes

As an Apprentice 1, you'll only work in two areas of Crate World. For now, you will be working on the X floor or on the Y elevator.

Your job as an Apprentice 1 is to move crates back and forth on the X floor or to move crates up and down the Y elevator.

All of the loading docks in Crate World can be found at locations marked with numbers from -10 to 0 to +10. You will also use numbers as directions, to move either right (+ direction) or left (- direction) on the X floor or up (+ direction) or down (- direction) on the Y elevator. Because floors and elevators don't stop at fractions, you will be working with numbers called **integers** by mathematicians.

Activity A. Start at the location of the green crate. Move the red crate in a **+ (positive sign)** direction to the right on the X level or up on the Y elevator to a new location.

You also will move crates in a **- (negative sign**) direction to the left on the X level or down on the Y elevator to new locations.

Watch out when you start working with negative numbers such as - 1 or - 8 as locations or directions. Which location is closer to 0? If you start at the location 8 with a direction of - 1, what location will your crate end up at? What happens if you start at location - 8?

For most of our lives we take for granted that numbers are positive, not negative. If you don't see a - (negative) sign in front of a number, it is a positive (+) location or telling you to move your crate a positive (+) direction.

Activity B. Check that you recognize the proper instruction to move from one crate to another. Are you going in a negative direction or in a positive direction? How far? Make sure you start at the correct (green) crate.

<u>Activity C</u>. You may have learned in the past that **+** means adding. Well, yes, we're making numbers greater when we add positive numbers.

But what if we have negative numbers? Here are some things that might happen.

- 8 + 8

0

Or

- 8 + 5

- 3

Or

- 8 + 10

2

Did you notice that that last example is like the subtraction we know?

- 8 + 10 is the same as 10 + (- 8) is the same as 10 - 8

Try writing instructions (math expressions) in different ways that mean the same thing.

Now see if you can use math to move your crates to the correct loading docks.

<u>Activity D</u>. Have you ever considered what a negative negative direction might be? You'll find out in Activity D.

The opposite of moving left is to move right; the opposite of moving right is to move left. The opposite of moving up is to move down; the opposite of moving down is to move up.

You might be asked to follow a direction such as - (- 1). You need to go to the opposite of the negative direction; you need to go in the positive direction. The opposite of moving - 1 is + 1, and so

- (- 1) is 1

[***Note for the future***: By the way, although you may not need this immediately, when you see something like - (- 1), you are also actually multiplying!

- (- 1) is the same as - 1 (- 1).

Therefore, if you multiply two negative numbers together, your answer is positive.

What if you multiply a positive and a negative number? Think about 1 (- 1), which is just -1.

Compare 1 (- 1) to - 1 (1). Are they really the same or different?

Remember, if something is confusing, you can always move numbers around to make them easier to work with. Just make sure that you keep their relationship the same.

1 (- 1) = - 1 (1) = - 1

Therefore, if you multiply a negative number with a positive number, your answer is negative. You will be using this knowledge when you become an Apprentice 4.]

Activity E. You have probably learned in the past that **-** means subtracting. But what is subtraction really? Now that you've been using negative numbers, you can see that subtracting can be seen as just one way of using negative numbers.

Look at this example.

$$3 - (- 5)$$

Do you treat - (- 5) the same as a negative negative number? Yes!

Are you subtracting? Not really.

$$3 - (- 5) = 3 + 5$$

Activity F. Sometimes a supervisor only wants to know how far you've moved crates, not your direction. The distance you move the crate is called the **absolute value**. In Activity F, you need to use absolute values. If you see vertical lines on each side of a number, you need to figure out the absolute value of that number. Here are two examples.

$$| 8 | = 8$$

$$| - 8 | = 8$$

If you've moved 8 spaces, whether you moved a positive or negative direction, you've moved 8 spaces. What is tricky is when you have to figure out the absolute value of a math expression that uses a calculation of several numbers. Just use what you've learned so far, then make the value of whatever is between the vertical lines positive. Here are two examples.

$$| 5 - 7 | = | - 2 | = 2$$

$$| - 10 + 4 | = | - 6 | = 6$$

When you finish all of the Apprentice I activities and are not losing crates or giving your boss the wrong information, you're ready to be an Apprentice 2!

Apprentice 2 Notes

As an Apprentice 2, you move crates all through Crate World. At the center of Crate World, you are at the intersection of floor X and elevator Y, shown by the number 0 for both the floor and the elevator. Mathematicians call a map that looks like Crate World a **coordinate graph**.

Activity A. You must recognize and find loading docks and crates at all locations. The locations are called **coordinates** and are written like this: **(x , y)**.

x and **y** can be any two integers in Crate World. (You CAN have coordinates that include fractions or decimals, but not in Crate World.) For example, the location of the center of Crate World is (0 , 0).

Find the **x coordinate** by finding the crate's location in relation to floor X (known to mathematicians as the **X axis**).

Find the **y coordinate** by finding where the crate is in relation to the Y elevator (known to mathematicians as the **Y axis**). Which floor of the Y elevator is at the same level as the crate?

Activity B. Check that you understand how to use coordinates by moving the crate to the correct location! If you could recognize the coordinates in Activity A, you should be able to put a crate in the correct location.

Activity C. Try to figure out the correct instructions to go from the crate at one loading dock to another. We are going to use **"rise"** to mean going up or down vertically, in a positive or negative direction. **"Run"** will mean going from side to side horizontally, in a positive or negative direction. Make sure you start at the correct crate (the green crate).

Activity D. Follow the directions from the green crate to take the red crate to the proper loading dock! If you were able to figure out Activity C, you should be able to move the red crate correctly in this activity.

<u>Activity E</u>. Ok, your boss wants to know how far you go as you travel between crates, travelling horizontally and vertically. Your boss doesn't want you to tell the directions you go, only the total distance, so you are going to be working with absolute values again.

You can count spaces, but try a math formula. It's easier to use math than to count spaces on the <u>Crate World</u> map if you are going large distances and using greater numbers. You will need to be comfortable working with absolute values for future activities also, so start now.

Here's an example. Say you have figured out the coordinates for your two crates**:** ***(- 10 , - 4)*** and ***(- 3 , - 5)***. Use this formula:

$$| x1 - x2 | + | y1 - y2 |$$

$$| - 10 - (- 3) | + | - 4 - (- 5) |$$

$$| - 10 + 3 | + | - 4 + 5 |$$

$$| - 7 | + | 1 |$$

$$7 + 1$$

$$8$$

You can see that it is very important to be very careful about writing down each coordinate and the appropriate + and **-** signs carefully in your math expression! It's easy to make a mistake and irritate your boss (and remain an Apprentice 2).

<u>Activity F</u>. You have never moved crates diagonally as an Apprentice 3, but as you finish your Apprentice 2 training, you need to start thinking about rise and run together as a **slope (rise / run).**

Slope is a measure of steepness. Mountains have slopes, and so do many routes in Crate World. Knowing the slope of routes in Crate World will take you from loading dock to loading dock faster and more easily in the future as you become an Apprentice 3.

Congratulations when you're ready to move on to Apprentice 3!

Apprentice 3 Notes

As an Apprentice 3, you will move crates around more efficiently, with less time and energy. (Maybe you will use more brain energy, but you will use less physical travel energy moving crates between locations.)

You will be using routes that go directly from one loading dock to another. A route can be used to send crates to any of the loading docks on its path. The instructions for these routes are called **linear equations**.

Linear means that you are talking about a straight line. (Yes, there are non-linear equations that are not straight lines, but Crate World Apprentices don't have a need for them.)

In an **equation**, there is an equal relationship between numbers or a math expression on one side of the equal sign and those on the other side. Equations can look many different ways, but whatever is on the left side will equal whatever is on the right side.

You will see linear equations that look different ways, depending on the information that you need to recognize or to make a particular route or line. By the time you go on to be an Apprentice 4, you will know how to read and make lines that go different directions and have different slopes.

Activity A. Some routes, which are either a **horizontal** (flat and level) or **vertical** (perpendicular to the horizon, going up and down) line, are described by a linear equation that might look like one of the two following examples.

$$x = 5$$

is the vertical line on which all loading docks have an x coordinate of 5! Every possible x is a 5. A crate on the route may have a y coordinate for a particular location on the line, but the line itself is just x = 5.

$$y = -3$$

is the horizontal line on which all loading docks have a y coordinate of - 3! Every possible y is a - 3.

Because these routes are horizontal or vertical, they are not diagonal. They are not described with a slope. Make sure you can recognize and make these routes even though we won't be using them in many other Crate World activities.

Activity B. **m** is the variable or name that Crate World mathematicians in the U.S. use for slope. Remember that the mathematical meaning of **slope is rise / run**. (Remember also that the **/ sign** can mean different things in different equations: division, fraction, ratio, etc.)

In this case, rise / run means how much the line "rises" (moves up or down) for the amount it "runs" (moves to left or right).

An amazing and helpful thing about math is that you can do the same old things with all sorts of numbers, whatever they mean in a particular situation. Sometimes you will have complicated slopes that you want to simplify so that you can work with easier or smaller numbers in an equation. When you see the / sign, you might divide rise by run. You might treat rise / run as a fraction that you can simplify. See what works to make numbers easier to work with. For example,

$$2 / 1 = 2$$

Just remember when you work with a slope of 2 that you're really talking about a rise / run of 2 / 1 .

Another example is 6 / 8.

$$6 / 8 = 3 / 4$$

It's quicker to make a line using a rise of 3 and run of 4 than by counting a rise of 6 and run of 8.

In this activity, you might notice that every route crosses the Y elevator at 0.

Mathematicians call where a line crosses the Y axis its **y-intercept**. In this activity, the y-intercept is always 0.

By the way, your boss will often leave out zeros in the equations for routes as you continue your apprenticeship. Every number means something, and every number that you expect but that is not there also means something. Keep alert!

At first in your Apprentice 3 training, you don't have to use math calculations to use the routes. However, soon you'll find that math calculations make using routes much easier. In fact, some things can't be done without a calculation, especially when you are an Apprentice 4, so start practicing now!

Activity C. This activity looks much more complicated than the last, so take a moment to look at the parts of the linear equations. If you break each equation down into its parts, it will make sense.

Your boss is using the y-intercept form of the red line route. The y-intercept form of a linear equation looks like this:

$$\mathbf{y = m x + b}$$

Any line that has a slope will have an equation basically like this. The parts of this equation shouldn't be so strange. You already have been working with **slope (m),** and you might remember that the **y-intercept (b)** is where a route crosses the y elevator (or y axis).

Remember **y** might be any y coordinate on a route**; x** might be any x coordinate on a route. So we don't have to say which x and which y to talk about a route. We do need to give x and y coordinate locations if we're talking about a particular location where, for instance, a crate might be or need to go.

Again, remember that m is the slope, which is rise / run.

Also, remember that b is the letter that mathematicians use to represent the y-intercept in the U.S. (Different countries may use different letters sometimes to represent numbers, but the math is the same.)

Activity E. Now check that you can use your understanding of your boss' instruction using a y-intercept form of a linear equation to find a route: **y = m x + b**

Look at this example:

$$y = (3/2)x - 6$$

This means you can start at the y -intercept of - 6 and imagine the line with a rise of 3 and run of 2 from that point. If you put your finger on a location on that line, you will have your answer confirmed: the route will appear.

You'll notice that the y - intercept is shown with a crate. Because that crate is already on the route, putting another crate on top of that crate or very near it will not make the route appear. As your boss says, that's too easy an answer!

Activity F. Now you're ready to start working with linear equations in different ways. Sometimes you know the linear equation, but you are interested in where a crate is. Sometimes you may only know part of the location of a crate.

Here's an example.

$$5 = x - 1$$

Because our boss has plugged specific numbers into the equation **y = m x + b :**

- we know the y coordinate of the crate: 5
- we know the y-intercept: - 1.
- we even know the slope, which has to be 1, which is the same as 1 / 1.

What we don't know is the x coordinate of the crate.

You can find the x coordinate by solving for x. You can probably figure out the answer by looking at the equation, but you can find the answer by isolating x on one side of the equation. You'll need to use this procedure when you start working with more complicated equations, so get enough practice.

Remember if you add something to one side of the equation, you need to add something to the other side of the equation. The same goes for subtracting, dividing, and multiplying. Be careful so you don't leave out any steps.

$$5 = x - 1$$

$$5 + 1 = x - 1 + 1$$

$$6 = x$$

OK, when you're understanding and using your Apprentice 3 skills well, you're ready to be an Apprentice 4!

Apprentice 4 Notes

As an Apprentice 4, you have much more responsibility! You must make sure that crates are where they are supposed to be and that they are moving where they need to go, even if your boss' instructions are messy or missing important information.

You will often receive packing instruction slips that have become smudged as they have been moved from hand to hand, level to level, and loading dock to loading dock. You will need to use math to figure out what the smudged numbers are.

Again you will most often be using the form of the equation for routes that is written

$$y = m x + b$$

(As Apprentice 5, you will see routes as linear equations written all sorts of ways, but if you are comfortable using the form y = m x + b, you can use it to make sense of other kinds of instructions.)

Activity A. Now your boss wants you to figure out a missing coordinate for a crate again. But this time your boss gives you one of the crate's coordinates and the slope. The y-intercept will always be 0 in this activity. (See how the equation y = m x + b is only y = m x in this activity, so b, the y-intercept, must be 0.)

Here's an example of how you can isolate a variable on one side of an equation to figure out the missing coordinate.

$$2 = (1 / 3) x$$

$$2 (3 / 1) = [(1 / 3) x] (3 / 1)$$

$$6 = x$$

Remember, if you multiply on one side of the equation, you have to multiply the same number on the other side of the equation. Do you see how we get rid of the 1 / 3 on the right side of the equation by multiplying it by 3 / 1? 1 / 3 multiplied by 3 / 1 equals 1!

You can often use the reciprocal (the reciprocal of 1 / 3 is 3 / 1) of a number to get rid of pesky fractions in equations.

One of the problems with using x as a variable is that it's easy to think x means multiplication. In algebra (which is what you're doing here, moving numbers around to figure out the values of variables), mathematicians use many different letters to stand for numbers. They also may use different kinds of parentheses and brackets to show which numbers go together. Here you are multiplying [(1 / 3) x] with (3 / 1). Take your time to make sense of equations. It's like reading a foreign language until you learn the language.

Activity B. Now your boss is giving you both slopes and y-intercepts, so sometimes your calculations will require a few more steps.

If y is smudged, you'll be able to find out what y is by finding the value of the other side of the equation.

For example, here's a packing slip with a smudged y.

$$y = (1/4)8 + 3$$

$$y = 8/4 + 3$$

$$y = 2 + 3$$

$$y = 5$$

If you identify the correct value of y, you'll see the crate at its (x, y) location of (8 , 5) and see the route described by: y = (1 / 4) x + 3, which includes all possible (x , y) loading docks on that route.

Activity C. If b (the y-intercept) is smudged, you'll need to find out what b is by isolating b on one side of the equation. Here's an example.

$$5 = (1/4)8 + b$$

$$5 = 8/4 + b$$

$$5 = 2 + b$$

$$5 - 2 = 2 + b - 2$$

$$3 = b$$

Activity D. If x is smudged, you may have to do a little more. Here's an example.

$$5 = (1/4)x + 3$$

$$5 - 3 = (1/4)x + 3 - 3$$

$$2(4/1) = [(1/4)x](4/1)$$

$$8 = x$$

Activity E. If the slope is missing, but you know where the route crosses the Y elevator (y-intercept), you can use the coordinates for x and y to figure it out. Here's an example.

$$5 = m(8) + 3$$

$$5 - 3 = m(8) + 3 - 3$$

$$2 = m(8)$$

$$2 / 8 = [m (8)] / 8$$

$$1 / 4 = m$$

<u>Activity F.</u> Now your boss wants you to figure out the slope for the route that includes crates at two locations. You can very efficiently decide on the route for other people to pick up crates if you can do this!

Use the formula: $m = (y1 - y2) / (x1 - x2)$

Example: slope for the route with crates at locations $(- 2 , - 4)$ and $(2 , - 1)$

$$m = (-4 - (- 1)) / (- 2 - (2))$$

$$m = - 3 / - 4$$

$$m = 3 / 4$$

Does it matter which x is x1 or which y is y2? Just be consistent. It's easy to make mistakes in calculating slopes if you don't write down coordinates and the equation correctly.

Note that you didn't have to figure out the equation for the route. You will do this as an Apprentice 5.

Congratulations when you are finding crates and routes accurately! You're ready to be an Apprentice 5.

Apprentice 5 Notes

As Apprentice 5, you start using numbers other than whole numbers, and you will also be going beyond Crate World! Maybe you've noticed that sometimes the lines for the routes go off beyond the Crate World grid. Well, those lines can go on forever even if we don't see them! But first....

Activity A. Your boss sometimes writes routes in forms other than y = m x + b. Move the numbers around so you can easily recognize the route. If you've been isolating variables in equations as an Apprentice 4, this should be easy if you're careful. Change each equation to the y = m x + b form.

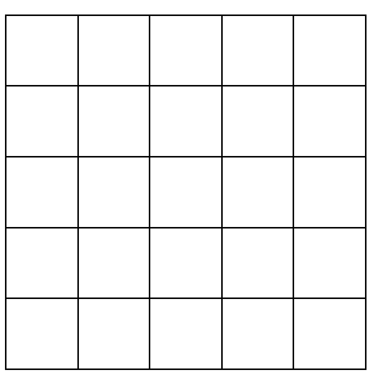
Activity B. In a future activity, to find distances between loading docks, you will need to be able to use squares of numbers and square roots of numbers.

In this activity, you will practice recognizing some common **squares of numbers**. Mathematicians also call squares of numbers other names. For example, you can read the following number in several ways.

$$\mathbf{5^2}$$

You can say "5 **squared**" or "5 **to the second power**" or "5 with **exponent 2**." Whichever way you say it, this means 5 times 5 or 25.

Why do we call these squares of numbers? Well, what if we have a square in which each side is 5 linear units?



We have 25 square units (otherwise known as the **area**) in all, of course.

[**By the way**, even though you don't need to use other exponents as an Apprentice 5 in Crate World, you also should know that you can use any number as a "power" or "exponent." Do you have an idea of what 5^3 is?

$$\boldsymbol{5^3}$$

$$\boldsymbol{5 \times 5 \times 5}$$

Mathematicians call values to the 3rd power **cubed** because this math expression shows the dimensions of a cube (5 x 5 x 5) and how many cubic units (125) would fill that cube (its **volume**).

Mathematicians also use exponents to talk about everything from probability (how often you might expect to pick a certain card from a deck, for instance) to the nature of the universe (how many dimensions best describe reality? probably not just 3).]

Activity C. Practice recognizing some common **square roots**.

If you know commonly used squares of numbers, then you will be able to recognize numbers for which you know square roots. To jump to the punch line, here's an example: the square root of 25 that you'll use as an Apprentice 5 is 5. (Hmm, 5^2 is 25.)

$$\sqrt{\boldsymbol{s}} = \sqrt{(\boldsymbol{n})(\boldsymbol{n})} = \boldsymbol{n}$$

$$\boldsymbol{so}$$

$$\sqrt{\mathbf{25}} = \sqrt{(\mathbf{5})(\mathbf{5})} = \mathbf{5}$$

(Note: A square root of a positive number can be a negative number [(- 5) (- 5) = 25)], but you will only use positive square roots as an Apprentice 5.)

Activity D. Use a formula devised by an ancient Crate World worker (well, actually he was a Greek mathematician and philosopher), Pythagoras, who measured distances by thinking of them as sides of a right triangle. Look up the **Pythagorean Theorem** to find out more about uses of this formula!

Here's an example when you know that crates are located at (- 2 , - 5) and (1, 4)

$$\sqrt{(\boldsymbol{y1} - \boldsymbol{y2})^2 + (\boldsymbol{x1} - \boldsymbol{x2})^2}$$

$$\sqrt{(\,-\mathbf{5} - \ \mathbf{4}\,)^2 + (\,-\mathbf{2}\ -\ \mathbf{1}\,)^2}$$

$$\sqrt{(\,-\mathbf{9}\,)^2 + (\,-\mathbf{3}\,)^2}$$

$$\sqrt{\mathbf{81} + \mathbf{9}}$$

$$\sqrt{\mathbf{90}}$$

$$\mathbf{9.5}$$

Note that the distances between loading docks are not often integers when you're working with diagonal lines. Use a calculator to find decimal answers. You can also make an estimate from the common roots you do know. For example, the square root of 90 is about the same distance from the square root of 81 and the square root of 100, so it's between 9 and 10, maybe about 9.5.

Activity E. You need to use math to find the equation for the route that goes through two crate locations. Your boss doesn't show you the Crate World map any more! If you're careful about every step and use everything you've learned so far, you can find linear equations from two points now.

Activity F. Finish your apprenticeship by sending a crate outside Crate World! Use numbers outside of -10 to 10. All you have to do is plug in coordinates from different possible crate locations and see which location is on your route!

Congratulations when you have finished your Crate World Apprenticeship!

Enjoy the wide world outside of Crate World. Find some interesting new things you can do with your skills!